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Clutch Friction Measurement

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Friction arises in nearly every application of mechanical engineering, either as a disturbance or as a useful effect. Its quantitative knowledge is required when the system behavior shall be influenced in a prescribed manner. Because mathematical models for friction that take all parameters into account do not exist, a direct and simple measurement strategy based on observer theory is proposed. To get realistic numerical results, the procedure is demonstrated for a vehicle clutch for which measured data is available. However, the procedure can be generalized to a wide field of application, such as mechanical brakes, which are also discussed in the paper.

Introduction

FRICTION plays an important role in nearly every application of mechanical engineering. Friction may cause severe difficulties in airplane landing gears due to landing impact and wheel spin up.[1] It leads to inaccuracy in fine point robot control.[2] Friction dampers, which are used to decrease vibration amplitudes of turbine blades in aircraft engines, may damage the turbine when sticking occurs.[3] Friction finally is used in any mechanical brake where it may cause undesired vibrations due to stick-slip processes.[4,5]

In some applications the frictional force must be known explicitly to overcome mechanical problems. This is the case in the aforementioned robot control as well as in any antiskid system for brakes or actively controlled landing gears. In all these cases, two or more (rigid) bodies are brought into contact with the aim of prescribing the necessary amount of pressure to get the desired friction reaction.

Since the first experimental results from the 17th and 18th century (Amonton, Euler, Coulomb), many attempts have been made to give a mathematical description of frictional processes.[6,7] It has been shown that Coulomb's law does not provide unique solutions in some cases[8] and that general mathematical models that include every important parameter have not been found until now.[6,9] If, however, friction influences the system state, then it should be possible to get exact information by an adequate treatment of corresponding state variable measurements.

This can be achieved using the observer theory.[10] Here, the observer generally yields good results when applied to the state equations that are augmented by a disturbance model.[11,12,13] In this paper this theory is used to determine the friction coefficient. Because the corresponding measurement contains the actual state of the system, it includes temperature and any other parameter that may be of influence. Therefore, the resulting friction coefficient may not only be treated as measurement output but may also be used for open-loop or closed-loop control. As a special application, a rotating friction clutch is considered. Such a device is used when an auxiliary engine is to be connected to a rotating shaft in a smooth manner. To get realistic simulations, a coupling between motor and drive train of a vehicle is considered for which experimental data is available.

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General Description

The considered mechanical model is represented by a multibody system (MBS) consisting of n bodies. Its basic dynamics may be described by the rather general equation

$$\sum_{i=1}^{n}\left\{\begin{pmatrix}\dfrac{\partial v_i}{\partial \dot{s}}\\[2mm] \dfrac{\partial \omega_i}{\partial \dot{s}}\end{pmatrix}^T \begin{bmatrix}(\dot{p}+\tilde{\bar{\omega}}p-f^e)_i\\ (\dot{L}+\tilde{\bar{\omega}}L-l^e)_i\end{bmatrix}\right\}=0 \tag{1}$$

where p and L are linear and angular momentum, respectively; $\tilde{\bar{\omega}}$ is the skew symmetric tensor providing vector product between $\bar{\omega}$ and p and L, and f^e and l^e are the impressed force and torque, respectively.[11]

Here, $\bar{\omega}$ means angular velocity of a chosen reference frame with respect to the inertial frame, whereas ω denotes the absolute angular velocity of a considered body, the components of which are to be calculated in that reference frame as well as those of the translational velocity v. Hence, Eq. (1) is valid for an arbitrarily moving reference frame. For example $\bar{\omega}=0$ for the inertial frame, and $\bar{\omega}=\omega$ for a body-fixed coordinate system.

Furthermore, $\dot{s}$ represents the vector of *minimal velocities,* being a regular combination of first-time derivatives of *minimal coordinates* z (generalized coordinates, Lagrangian coordinates). (Minimal velocities need not be integrable with respect to time but correspond to quasicoordinates in general.[7])

Subsystem Equations

In considering MBS with r rigid bodies coming into frictional contact, it will be useful to formulate Eq. (1) in terms of r subsystems. This is easily obtained by splitting up the sum in Eq. (1) and using the chain rule of differentiation,

$$\sum_{k=1}^{r}\left(\frac{\partial \dot{s}_k}{\partial \dot{s}}\right)^T\sum_{j=1}^{mk}\left\{\begin{pmatrix}\dfrac{\partial v_j}{\partial \dot{s}_k}\\[2mm] \dfrac{\partial \omega_j}{\partial \dot{s}_k}\end{pmatrix}^T \begin{bmatrix}(\dot{p}+\tilde{\bar{\omega}}p-f^e)_i\\ (\dot{L}+\tilde{\bar{\omega}}L-l^e)_i\end{bmatrix}\right\}_k=0 \tag{2}$$

Here, r is the number of subsystems, each consisting of m_k bodies. The corresponding velocities $\dot{s}_k$, $k=1(1)r$, are chosen independent of each other.

Equations of External Variables

We assume that the inner relative motions of the subsystems will not affect the output gross motion in a significant way. The corresponding variables of relative motion are called *internal variables,* and the remaining gross motion is assigned to the *external variables* that can be seen and measured from "outside" of the system. Only the external variables will be retained in the following. The number of degrees of freedom is then equal to the number r of considered subsystems. (It is evident that the separation of state variables into independent internal and external ones is based on the assumption of negligible coupling.) The simplest case to be considered is a mechanical system that consists of only two subsystems with a frictional torque at the interface. Such models describe the general case of mechanical brakes or friction clutches. Because measured data of a car coupling has been available, an example of vehicle dynamics according to Fig. 1 will be numerically treated. Here, the aim is to adjust the angular velocity of the gearbox input and of the motor output. However, the procedure to determine the friction coefficient between the contacting bodies is evidently the same for a mechanical brake (Fig. 2) where to avoid locking the contacting bodies must be kept sliding with minimum slip to obtain maximum braking energy. In addition, remarks on friction brakes will be given.

Clutch Equations

Equations of Motion

In the case of a planar motion of a drive assembly according to Fig. 1, Eq. (2) yields

$$\left[\left(\frac{\partial v}{\partial \dot{s}}\right)^T m_M\left(\frac{\partial v}{\partial \dot{s}}\right) + \left(\frac{\partial \Omega_M}{\partial \dot{s}}\right)^T J_M\left(\frac{\partial \Omega_M}{\partial \dot{s}}\right)\right]_{\text{Motor}} \ddot{s} - \left(\frac{\partial \Omega_M}{\partial \dot{s}}\right)^T l_M$$
$$+ \left\{\sum_{\text{Gear}}\left[\left(\frac{\partial v_i}{\partial \dot{s}}\right)^T m_i\left(\frac{\partial v_i}{\partial \dot{s}}\right) + \left(\frac{\partial \Omega_i}{\partial \dot{s}}\right)^T J_i\left(\frac{\partial \Omega_i}{\partial \dot{s}}\right)\right]\right\}\ddot{s}$$
$$- \left(\frac{\partial \Omega_K}{\partial \dot{s}}\right)^T l_K + \left\{\sum_{\text{Car}}\left[\left(\frac{\partial v}{\partial \dot{s}}\right)^T m_i\left(\frac{\partial v}{\partial \dot{s}}\right)\right]\right.$$
$$\left. + 4\left(\frac{\partial \Omega_R}{\partial \dot{s}}\right)^T J_R\left(\frac{\partial \Omega_R}{\partial \dot{s}}\right)\right\}\ddot{s} - \left(\frac{\partial \Omega_R}{\partial \dot{s}}\right)^T R f_w = 0 \tag{3}$$

where

$$\dot{s} = (\Omega_M, \Omega_K)^T$$
$$\frac{\partial \Omega_i}{\partial \Omega_j} = i_{ij}, \qquad \forall i,j \neq M,K$$
$$\frac{\partial \Omega_M}{\partial \Omega_K} = 0, \qquad \frac{\partial \Omega_K}{\partial \Omega_M} = 0 \tag{4}$$

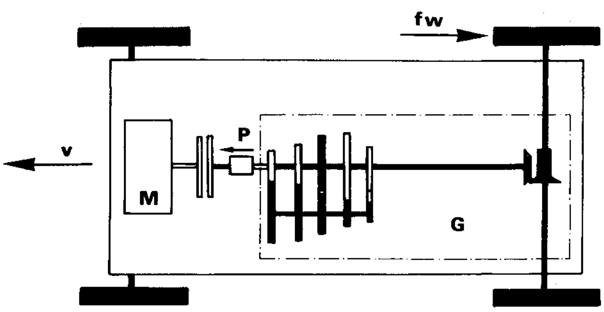

Fig. 1 Drive assembly (schematic, M: motor, G: gear).

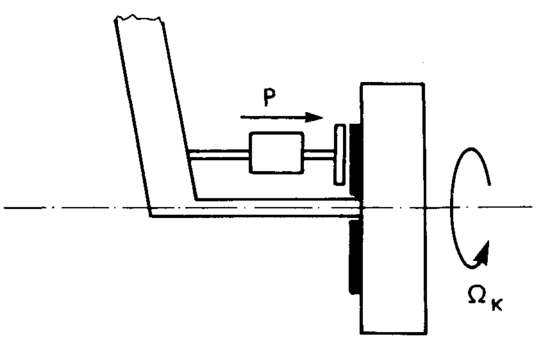

Fig. 2 Mechanical brake (schematic).

where index M is for the motor, index K is for the clutch, and $i_{i,j}$ represent constant transmission ratios. Equation (4) holds if $\Delta\Omega = \Omega_M - \Omega_K$ is not equal to zero, i.e., where there is slip at the clutch interface. Otherwise

$$\dot{s} = \Omega_M = \Omega_K$$
$$\frac{\partial \Omega_M}{\partial \Omega_K} = 1$$
$$\frac{\partial \Omega_K}{\partial \Omega_M} = 1 \tag{5}$$

Then, the actual clutch torque corresponds to a constraint torque.

The governing equations may be summarized to

$$\begin{bmatrix} J + J_M\left(\frac{\partial \Omega_M}{\partial \Omega_K}\right)^2 & 0 \\ 0 & J_M + J\left(\frac{\partial \Omega_K}{\partial \Omega_M}\right)^2 \end{bmatrix} \begin{pmatrix} \dot{\Omega}_K \\ \dot{\Omega}_M \end{pmatrix}$$
$$= \begin{pmatrix} 1 - \frac{\partial \Omega_M}{\partial \Omega_K} \\ \frac{\partial \Omega_K}{\partial \Omega_M} - 1 \end{pmatrix} M_K + \begin{pmatrix} -1 \\ -\frac{\partial \Omega_K}{\partial \Omega_M} \end{pmatrix} M_W + \begin{pmatrix} \frac{\partial \Omega_M}{\partial \Omega_K} \\ 1 \end{pmatrix} M_M \tag{6}$$

Here, J_M represents the motor moment of inertia, and J is the moment of inertia of the whole equipment, excluding the motor moment of inertia, which may be expressed as

$$J = \sum_{i=1}^{N} J_i\left(\frac{\partial \Omega_i}{\partial \Omega_K}\right)^2 + m_V R^2\left(\frac{\partial \Omega_R}{\partial \Omega_k}\right)^2:$$

R: wheel radius

Ω_R: wheel angular velocity

m_V: total vehicle mass

N: number of rotating bodies, excluding motor

M_K: clutch torque

$M_W = \sum_{i=1}^{4}\left(\frac{\partial \Omega_R}{\partial \Omega_K}\right) R f_w$: resistance torque

f_w: resistance force acting at wheel contact

$M_M = M_{M0} + k\Omega_M$: motor torque

Equation (6) holds for every state of the system. If $\Delta\Omega$ has a definite value, Eq. (4) represents two degrees of freedom. If, on the other hand, $\Delta\Omega$ becomes zero, both scalar differential equations represented by Eq. (6) become identical and M_K vanishes as a nonactive constraint torque. In the latter case, the value of M_K is automatically given by an implicit equilibrium condition.

Clutch Torque

In considering dry friction, it is—by experience—known that the friction resistance depends on the normal force and the relative velocity Δv. The amount of friction is then characterized by a velocity dependent coefficient $\bar{\mu}$. This coefficient, however, depends on material and facing of the contacting bodies as well as on lubrication and temperature and may alter during operational processes. Various measurements have been carried out, but no analytical formula exists. The common way to describe the friction coefficient is by curve fitting of measurements. We will use this procedure here, too. Because the transition from stiction to sliding friction is smooth, we assume

$$\frac{d\bar{\mu}}{d|\Delta v|} = -T\bar{\mu} \tag{7}$$

with T describing the transition from stiction to sliding friction, and

$$\begin{pmatrix} \Delta v \to 0 : \bar{\mu} = \bar{\mu}_\infty - \Delta\bar{\mu} \\ \Delta v \to \infty : \bar{\mu} = \bar{\mu}_\infty \end{pmatrix} \tag{8}$$

In the case of dry friction, $\Delta\bar{\mu} = \bar{\mu}_\infty - \bar{\mu}_0$ is a negative scalar. When liquid-filled couplings are considered, $\Delta\bar{\mu}$ is positive for sufficiently large $\Delta v > 0$. Recently, couplings have been considered that use a special paper lining between the contacting bodies where $\Delta\bar{\mu}$ is positive for every value of $\Delta v > 0$. In this case, slip-stick phenomena are avoided when approaching $\Delta v \to 0$. Unfortunately, no publications on this kind of facing material exist because it is still being concealed by industry.

In considering the clutch behavior, Eq. (7) is examined for two rotating disks coming into contact. Then, the momentary relative velocity at a certain point (r,ϕ) of one of the disks is

$$\Delta v = r\Delta\Omega \tag{9}$$

where $\Delta\Omega$ is the difference of angular speed of the contacting disks with radius R_D. The corresponding frictional torque then reads

$$\begin{aligned} |M_K| &= \int_0^{2\pi}\int_0^{R_D}\left\{p\rho^2\bar{\mu}_\infty\left[1-\left(\frac{\Delta\bar{\mu}}{\bar{\mu}_\infty}\right)\exp(-T\rho|\Delta\Omega|)\right]\right\} d\rho\, d\phi \\ &= pA\frac{R_D}{3}\left\{\bar{\mu}_\infty + 3\Delta\bar{\mu}\left[\frac{(x^2+2x+2)e^{-x}-2}{x^3}\right]\right\} \end{aligned} \tag{10}$$

where

$A = 2\pi R_D^2$ (clutch disk area)

$x = TR_D|\Delta\Omega|$

p = clutch pressure

A calculation of limiting values $(\Delta\Omega \to 0,\infty)$ yields

$$\begin{aligned} |M_K| &= pA\frac{R_D}{3}\left\{\bar{\mu}_\infty + 3\Delta\bar{\mu}\frac{(d/dx)[(x^2+2x+2)e^{-x}-2]}{(d/dx)(x^3)}\right\}_{\Delta\Omega=0,\infty} \\ &= p\left\{A\frac{R_D}{3}[\bar{\mu}_\infty - \Delta\bar{\mu}\exp(-TR_D|\Delta\Omega|)]_{\Delta\Omega=0,\infty}\right\} \stackrel{\text{def}}{=} p\mu \end{aligned} \tag{11}$$

To simplify numerical calculations, this equation will be used as an approximation for finite values of $\Delta\Omega$ instead of Eq. (10). There is no significant difference, as can be seen in Table 1 [with $\mu = A(R_D/3)\bar{\mu}$, $TR_D = 333$].

The result is finally verified by measurements. The corresponding experimental result from curve fitting is

$$|M_K| = p\mu, \quad \mu = \mu_o + \frac{|\Delta\Omega|}{a|\Delta\Omega| + b}, \quad \mu_o = \mu_\infty - \Delta\mu \tag{12}$$

With $1/a = \Delta\mu$, $1/b = TR_D\Delta\mu$, Eq. (12) represents a first-order Taylor expansion of Eq. (11). Here, measured data are $\mu_o = 6/100$, $a = 100/3$, and $b = 1/10$.

It should be emphasized that the clutch torque according to Eq. (11) is verified by measurements that correspond to a given set of parameters like material, surface, and temperature. As mentioned earlier, the torque differs when these parameters vary. However, this is insiginficant in the following, because Eq. (11) is only used as a nominal input for computer simulation and remains "unknown" for the observer, which will be discussed next.

Measurement and Estimation

Clutch Friction Coefficient

It is assumed that Ω_M will be measured. The corresponding measurement equation then reads

$$y_M = (0\ 1)s, \qquad y_M \in \mathbb{R}^1 \tag{13}$$

In case of $\Delta\Omega \neq 0$ ($\partial\Omega_M/\partial\Omega_K = 0$, $\partial\Omega_K/\partial\Omega_M = 0$), the motor differential equation [variable Ω_M in Eq. (6)] is used for estimation:

$$\dot{\Omega}_M = \frac{k}{J_M}\Omega_M - \frac{1}{J_M}M_K + \frac{1}{J_M}M_{M0} \tag{14}$$

where

$$M_K = \left[1 - \left(\frac{\Delta\mu}{\mu_\infty}\right)\exp(-TR_D|\Delta\Omega|)\right]p\mu_\infty \operatorname{sign}\Delta\Omega \tag{15}$$

If the pressure p is kept constant during a time interval ΔT,

$$n\Delta T \le t \le (n+1)\Delta T : p = \text{const} \tag{16}$$

then the actual friction coefficient may be estimated using

$$\dot{y} = \kappa y + f$$

with

$$y = \Omega_M + \frac{M_{M0}}{k}, \quad \kappa = \frac{k}{J_M}, \quad f = -\frac{\mu}{J_M}p \tag{17}$$

The inhomogeneous term of Eq. (17) is approximated in the next step by a polynomial of $(r-1)$th order with respect to time,

$$f = e_1^T z, \quad \dot{z} = Fz, \quad F = \begin{pmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 1 \\ 0 & \dots & & 0 \end{pmatrix}$$

$$\in \mathbb{R}^{r,r}, \qquad e_1^T = (1, 0, 0, \dots, 0) \in \mathbb{R}^r \tag{18}$$

and yields

$$\begin{pmatrix} \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \kappa & e_1^T \\ 0 & F \end{pmatrix}\begin{pmatrix} y \\ z \end{pmatrix} \Rightarrow \dot{x} = Ax \tag{19}$$

Equation (19) shows that an estimation of f using Eq. (18) is possible because the controllability condition

$$\operatorname{Rank}[e_1 | F^T e_1 | \cdots | F^{(r-1)^T} e_1] = r \tag{20}$$

holds for every dimension r of Eq. (18).[11]

The simplest case that will be considered in the following is $r = 1$, $(F \equiv 0)$.[2] Then, the basic observer equations[10] for the

Table 1 Different dry friction coefficients

$\Delta\Omega$	10^{-6}	3×10^{-4}	3×10^{-2}
$M_K(11)$: μ =	0.060	0.0629	0.090
$M_K(10)$: μ =	0.060	0.0622	0.090

estimation $\hat{x}$ of the state variables x and the nonmeasured state subspace, characterized by matrix T,

$$(x-\hat{x})\rightarrow 0, \qquad (\xi - Tx)\rightarrow 0 \tag{21}$$

will be fulfilled with

$$\dot{\xi} = -\lambda\xi + Ly_M \tag{22}$$

$$\hat{x} = S_1\xi + S_2 y_M \tag{23}$$

$$\xi = \xi \in I\!R^1, \quad L = L \in I\!R^1, \quad y_M = y \in I\!R^1, \hat{x} \in I\!R^2 \tag{24}$$

Choosing

$$T = (-\lambda \;\; 1), \quad S_1 = (0 \;\; 1)^T, \quad S_2 = (1 \;\; \lambda)^T \tag{25}$$

yields

$$(x - \hat{x}) = \begin{pmatrix} 0 \\ z - \xi - \lambda y \end{pmatrix} \rightarrow 0 \tag{26}$$

$$(\dot{\xi} - T\dot{x}) = -\lambda\xi + Ly + \lambda(z + \kappa y)$$

$$= \lambda(z - \xi) + (\lambda\kappa + L)y \rightarrow 0 \tag{27}$$

One can directly use the asymptotic value of Eq. (26) in Eq. (27) by augmenting with $-\lambda^2 y + \lambda^2 y$ (underlined) to obtain L:

$$(\dot{\xi} - T\dot{x}) = \underbrace{\lambda(z - \xi - \underline{\lambda y})}_{\rightarrow 0} + [\underline{\lambda(\lambda} + \kappa) + L]y \rightarrow 0$$

$$\Rightarrow L = -\lambda(\lambda + \kappa) \tag{28}$$

Thus, Eq. (23) becomes

$$\hat{x} = (\hat{y} \;\; \hat{z})^T = [y \quad \lambda(\alpha + y)]^T \tag{29}$$

with $\alpha = \xi/\lambda$ and

$$\begin{pmatrix} \dot{y} \\ \dot{\alpha} \end{pmatrix} = \begin{pmatrix} \kappa & 0 \\ -(\lambda + \kappa) & -\lambda \end{pmatrix} \begin{pmatrix} y \\ \alpha \end{pmatrix} + \begin{pmatrix} f \\ 0 \end{pmatrix} \tag{30}$$

from Eqs. (17) and (22).

The estimation, Eq. (18), is then

$$\hat{f} = e_1^T \hat{z} = \lambda(\alpha + y) \tag{31}$$

To give an idea of the obtained estimation quality, Eq. (30) may be transformed using

$$\bar{x} = \begin{pmatrix} 1 & 0 \\ \lambda & \lambda \end{pmatrix} \begin{pmatrix} y \\ \alpha \end{pmatrix} = \begin{pmatrix} y \\ \hat{f} \end{pmatrix} \tag{32}$$

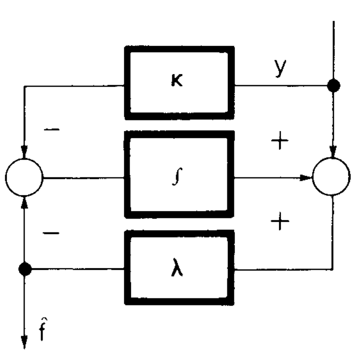

Fig. 3 Clutch pressure time history, p(N × 0.09/m²) over time (s).

Table 2 Estimation of different μ_∞ values

	M_{M0}	M_W	μ_∞	μ_∞	μ_∞
Exact	320.00	300.00	0.07	0.08	0.09
Estimated	320.0001	299.9993	0.06999	0.07999	0.09001

$$\Rightarrow \begin{pmatrix} \dot{y} \\ \dot{\hat{f}} \end{pmatrix} = \begin{pmatrix} \kappa & 0 \\ 0 & -\lambda \end{pmatrix} \begin{pmatrix} y \\ \hat{f} \end{pmatrix} + \begin{pmatrix} 1 \\ \lambda \end{pmatrix} f$$

$$\Rightarrow \begin{bmatrix} y(t) \\ \hat{f}(t) \end{bmatrix} = \begin{pmatrix} y_0 e^{\kappa t} \\ \hat{f}_0 e^{-\lambda t} \end{pmatrix} + \begin{bmatrix} e^{\kappa t} \int_0^t f(\tau) e^{-\kappa\tau} \, d\tau \\ e^{-\lambda t} \int_0^t \lambda f(\tau) e^{\lambda\tau} \, d\tau \end{bmatrix} \tag{33}$$

Assuming $|\Delta\Omega| = ct + \Delta\Omega_0$ for t such that $0 \le t \le \Delta T$ and an arbitrary constant c yields

$$\hat{f} = \left(\hat{f}_0 - \frac{1}{J_M}\left\{M_{M0} - p\mu_\infty\left[1 - \left(\frac{\Delta\mu}{\mu_\infty}\right)\left(\frac{\lambda}{\lambda - TR_D c}\right)\right]\right\}\right)e^{-\lambda t}$$

$$+ \frac{1}{J_M}\left\{M_{M0} - p\mu_\infty\left[1 - \left(\frac{\Delta\mu}{\mu_\infty}\right)\right.\right.$$

$$\left.\left.\times \left(\frac{\lambda}{\lambda - TR_D c}\right)\exp(-TR_D|\Delta\Omega|)\right]\right\} \tag{34}$$

which approximates the exact term

$$\frac{1}{J_M}\left\{M_{M0} - p\mu_\infty\left[1 - \left(\frac{\Delta\mu}{\mu_\infty}\right)\exp(-TR_D|\Delta\Omega|)\right]\right\} \tag{35}$$

if λ is sufficiently large. To damp the initial error up to 99% within $\Delta T = 10$ ms, a value of $\lambda \simeq 500$ is needed.

First investigations are made with $\mu \simeq$ const. This means that by inserting M_K according to Eq. (11), one starts with relatively high values of $|\Delta\Omega|$. Starting with p_1, one gets

$$\hat{f}_1 = \frac{1}{J_M}(M_{M0} - p_1\mu_\infty) \tag{36}$$

Thus, using two measurements assigned to p_1 and p_2 yields

$$\mu_\infty = \left(\frac{\hat{f}_1 - \hat{f}_2}{p_2 - p_1}\right)J_M, \qquad p_2 \neq p_1$$

$$M_{M0} = J_M\hat{f}_1 + p_1\mu_\infty$$

$$M_W = -J\left[\frac{\Omega(p_2) - \Omega(p_1)}{\Delta T}\right] + p_2\mu_\infty \tag{37}$$

where M_W [from Eq. (6)] is assumed to be constant in ΔT. Three different values for μ_∞ were estimated for a given time history of pressure. The results are shown in Table 2.

Of course, the outlined procedure is not restricted to μ_∞ but can also be calculated for every time interval ΔT where p is kept constant (see Fig. 3).

Detuning—Friction and Stiction

Although a linear approximation of $M_M = M_{M0} + k\Omega_M$ will hold in general, there must be sufficient information on k. This is easily demonstrated by Eq. (33). Let $\kappa(\kappa = k/J_M)$ change to $\bar{\kappa}$. Then, the transformation according to Eq. (32) yields

$$\begin{pmatrix} \dot{\Omega}_M \\ \dot{\hat{f}} \end{pmatrix} = \begin{pmatrix} \bar{\kappa} & 0 \\ \lambda\Delta\bar{\kappa} & -\lambda \end{pmatrix} \begin{pmatrix} \Omega_M \\ \hat{f} \end{pmatrix} + \begin{pmatrix} 1 \\ \lambda \end{pmatrix} f \tag{38}$$

1) If Eq. (38) is not tuned with $\Delta\bar{\kappa}\to 0$, then an additional error $\Delta\hat{f}$ arises. This error is, for f = const,

$$\hat{f}\to\hat{f}+\Delta\hat{f}$$

$$\hat{f}=\hat{f}_0 e^{-\lambda t}+f(1-e^{-\lambda t})$$

$$\Delta\hat{f}=\Delta\bar{\kappa}\left[\left(\frac{\lambda}{\lambda+\bar{\kappa}}\right)\left(\Omega_{M0}+\frac{f}{\bar{\kappa}}\right)(e^{\bar{\kappa}t}-e^{-\lambda t})+\frac{f}{\bar{\kappa}}(e^{-\lambda t}-1)\right] \quad (39)$$

It is clear that $\Delta\bar{\kappa}$ should be kept small to get a good estimate.

2) If only a measurement device is considered instead of an on-line estimation in a vehicle, for instance, a separate motor control that keeps the motor torque constant will be suitable.

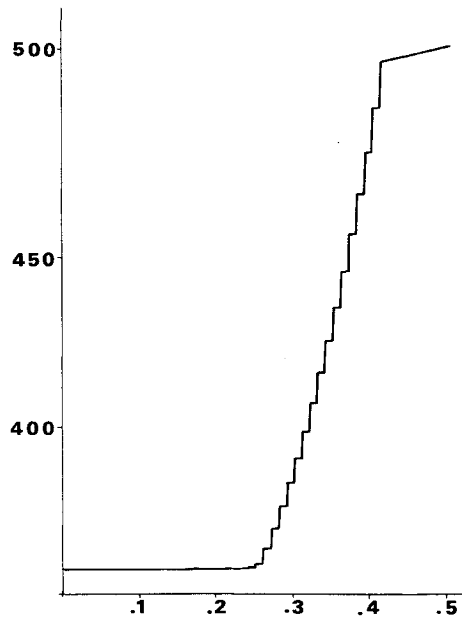

Fig. 4 Estimation.

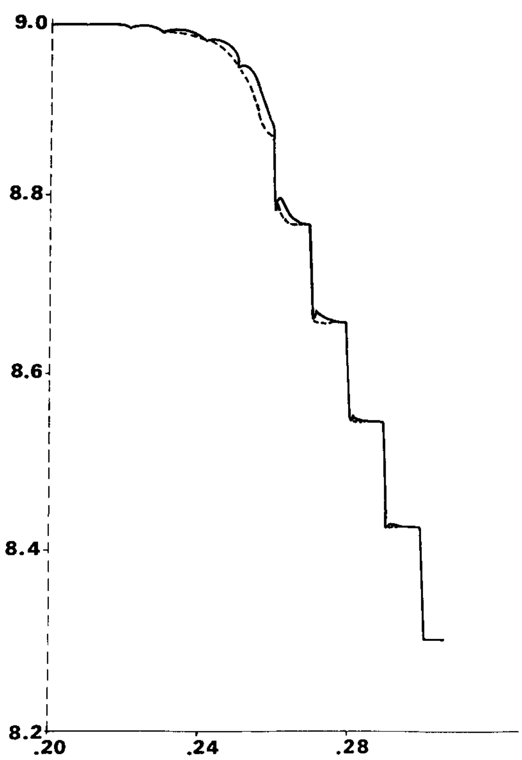

Fig. 5 Friction coefficient (10^{-2}) over time (s); dashed line: nominal input, solid line: estimation.

Table 3 Numerical values

Motor	M_{M0}, Nm 320	k, Nms 0.17	J_M, kgm^2 0.15
Clutch ($\mu\sim A\frac{R_D}{3}\bar{\mu}$)	μ_∞, m^3 0.09	$\Delta\mu$, m^3 0.03	TR_D, s 333
Car	M_W, Nm 300	J, kgm^2 15	

This means $\kappa = 0$ and simplifies the estimation strategy (see Fig. 3).

3) Once the coupling is locked due to stiction, the observer is automatically detuned. Inserting $(\partial\Omega_M/\partial\Omega_K)=1$ into Eq. (6) yields along with Eq. (38)

$$\bar{\kappa}=\frac{k}{J+J_M}$$

$$\Delta\bar{\kappa}=\frac{k}{J+J_M}-\frac{k}{J_M}$$

$$\hat{f}=\frac{M_{M0}-M_W}{J+J_M} \quad (40)$$

Then, the estimate no longer contains any information on the friction coefficient. This is due to the fact that the clutch torque M_K vanishes as a constraint torque in Eq. (6). This abrupt qualitative change indicates stiction.

Friction Coefficient in Mechanical Brakes

In the case of a friction brake, Eq. (6) reduces to the first component because there is no motor dynamics: $\Omega_M\equiv 0$. The remaining variable Ω_K is then assigned to the wheel angular velocity. Variables that are used in Eq. (17) have to be replaced by

$$y=\Omega_K+\frac{M_W}{J},\quad \kappa\equiv 0,\quad f=-\frac{\mu}{J}p \quad (41)$$

The estimation procedure reduces to a very simple and unsensitive process (see Fig. 3 with $\kappa\equiv 0$).

Numerical Results

To demonstrate the outlined theory, numerical values given in Table 3 have been used for calculation of the vehicle clutch. The clutch is driven with a prescribed pressure time history as demonstrated in Fig. 4. (This time history is from an optimal closed-loop control using pole placement. The only value that is fed back is the motor angular velocity, $p=\beta\Omega_M/\hat{\mu}$, where β is obtained from optimization and $\hat{\mu}$ represents the friction coefficient estimate. This kind of closed-loop control slows down Ω_M smoothly and avoids jerklike disturbances of car velocity.) The pressure is kept constant within a control time interval of $\Delta T = 10$ ms. The friction coefficient is depicted in Fig. 5, where the dashed line corresponds to the nominal input and the solid line shows the estimate. At the end of every estimation interval ΔT the estimation error is less than 0.04%.

Conclusions

Friction arises in nearly every application of mechanical engineering, as an undesired disturbance on the one hand or as a useful effect such as with mechanical brakes or couplings on the other hand. In any case, the actual friction coefficient must be known to achieve optimal behavior. Here, an application of the observer theory results in an extremely simple measurement strategy that gives direct information on the friction coefficient from a state variable measurement. To get realistic numerical results, the procedure is calculated for a rotating clutch of a vehicle where measured friction data have been available. Some remarks are added to the problem of friction brakes, where the proposed procedure is even simpler (just one amplifier and one integrator is needed) and totally unsensitive with respect to dynamical effects.

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